Back-Reaction: A Cosmological Panacea

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We present a solution to the dark energy problem in terms of the Effective Energy Momentum Tensor (EMT) of cosmological perturbations. This approach makes use of the gravitational back-reaction of long wavelength (super-Hubble) fluctuation modes on the background metric. Our results indicate that, following preheating, the energy density associated with back-reaction is sub-dominant and behaves as a tracker during the radiation era. At the onset of matter domination, however, the effects of back-reaction begin to grow relative to the matter density and the associated equation of state quickly approaches that of a cosmological constant. Using standard values for the preheating temperature and the amplitude of the inflaton following preheating, we show that this mechanism leads to a very natural explanation of dark energy. We comment on other recent attempts to explain the dark energy using back-reaction and their relation to our work.

I. INTRODUCTION

The nature and origin of dark energy stand out as two of the great unsolved mysteries of cosmology. Two of the more popular explanations are either a cosmological constant Λ, or a new, slowly rolling scalar field (a quintessence field). If the solution of the dark energy problem proved to be a cosmological constant, one would have to explain why it is not 120 orders of magnitude larger (as would be expected in a non-supersymmetric field theory), nor exactly zero (as it would be if some hidden symmetry were responsible for the solution of the cosmological constant problem), and why it has become dominant only recently in the history of the universe. These are the "old" and "new" cosmological constant problems in the parlance of [1]. To date, this has not been accomplished satisfactorily, despite intensive efforts. If, instead of Λ, the solution rested on quintessence, one would need to justify the existence of the new scalar fields with the finely tuned properties required of a quintessence field (e.g. a tiny mass of about 10^{-33}eV if the field is a standard scalar field). Clearly, both of the above approaches to explaining dark energy lead directly to serious, new cosmological problems. In this paper, we will explore an approach to explaining dark energy which does not require us to postulate any new matter fields.

There exist tight constraints on Λ from various sources - Big Bang Nucleosynthesis (BBN) [2], cosmic microwave background (CMB) anisotropies [3], cosmological structure formation [4] - which rule out models where the vacuum energy density is comparable to the matter/radiation energy density at the relevant cosmological times in the past. However, it could still be hoped that a variable Λ model might be compatible with observation since the value of ρ_Λ is constrained only for certain redshifts. In fact, the above constraints taken together with the results from recent supernovae observations [5],[6] leads one to posit that the vacuum energy density might be evolving in time.

This leads directly to the proposal of tracking quintessence [7]. However, some of the drawbacks of quintessence were mentioned above. A preferable solution would combine the better features of both quintessence and a cosmological constant: a tracking cosmological "constant".

In this letter, we discuss the possibility that the energy-momentum tensor of long wavelength cosmological perturbations might provide an explanation of dark energy. The role of such perturbations in terminating inflation and relaxing the bare cosmological constant was investigated some time ago in [8, 9] (see also [10]). However, this mechanism can only set in if the number of e-foldings of inflation is many orders of magnitude larger than the number required in order to solve the horizon and flatness problems of Standard Big Bang cosmology. Here, we are interested in inflationary models with a more modest number of e-foldings. We discover that, in this context, the EMT of long wavelength cosmological perturbations results in a tracking cosmological "constant" of purely gravitational origin and can be used to solve the "new" cosmological constant problem.

We begin by reviewing the formalism of the effective EMT of cosmological perturbations in Section 2. We recall how, in the context of slow-roll inflation, it could solve the graceful exit problem of certain inflationary models. We then extend these results beyond the context of slow-roll inflation in Section 3. In Section 4, we investigate the behaviour of the EMT during the radiation era and show that the associated energy density is sub-dominant and tracks the cosmic fluid. We examine the case of the matter era and show how the EMT can solve the dark energy problem in section 5. In Section 6 we consider the effects of back-reaction on the scalar field dynamics. We then summarize our results and comment on other attempts to use the gravitational back-reaction of long wavelength fluctuations to explain dark energy.

II. THE EMT

The study of effective energy-momentum tensors for gravitational perturbations is not new [11, 12]. The interests of these early authors revolved around the effects of high-frequency gravitational waves. More recently, these methods were applied [8, 9] to the study of the effects of long-wavelength scalar metric perturbations and its application to inflationary cosmology.

The starting point was the Einstein equations in a background defined by

$$
\begin{aligned}
ds^2 &= a^2(\eta)((1 + 2\Phi(x,\eta))d\eta^2 \\
&- (1 - 2\Phi(x,\eta))(\delta_{ij}dx^i dx^j))
\end{aligned}
\tag{1}
$$

where η is conformal time, $a(\eta)$ is the cosmological scale factor, and $\Phi(x,\eta)$ represents the scalar perturbations (in a model without anisotropic stress). We are using longitudinal gauge (see e.g. [13] for a review of the theory of cosmological fluctuations, and [14] for a pedagogical overview). Matter is, for simplicity, treated as a scalar field φ.

The modus operandi of [8] consisted of expanding both the Einstein and energy-momentum tensor in metric (Φ) and matter ($\delta\varphi$) perturbations up to second order. The linear equations were assumed to be satisfied, and the remnants were spatially averaged, providing the equation for a new background metric which takes into account the back-reaction effect of linear fluctuations computed up to quadratic order

$$
G_{\mu\nu} = 8\pi G \left[T_{\mu\nu} + \tau_{\mu\nu} \right],
\tag{2}
$$

where $\tau_{\mu\nu}$ (consisting of terms quadratic in metric and matter fluctuations) is called the effective EMT.

The effective energy momentum tensor, $\tau_{\mu\nu}$, was found to be

$$
\begin{aligned}
\tau_{00} &= \frac{1}{8\pi G} \left[+12H\langle\phi\dot{\phi}\rangle - 3\langle(\dot{\phi})^2\rangle + 9a^{-2}\langle(\nabla\phi)^2\rangle \right] \\
&+ \langle(\delta\dot{\varphi})^2\rangle + a^{-2}\langle(\nabla\delta\varphi)^2\rangle \\
&+ \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle \quad,
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
\tau_{ij} &= a^2\delta_{ij}\left\{ \frac{1}{8\pi G} \left[(24H^2 + 16\dot{H})\langle\phi^2\rangle + 24H\langle\dot{\phi}\phi\rangle \right. \right. \\
&+ \left. \langle(\dot{\phi})^2\rangle + 4\langle\phi\ddot{\phi}\rangle - \frac{4}{3}a^{-2}\langle(\nabla\phi)^2\rangle \right] + 4\dot{\varphi_0}^2\langle\phi^2\rangle \\
&+ \langle(\delta\dot{\varphi})^2\rangle - a^{-2}\langle(\nabla\delta\varphi)^2\rangle - 4\dot{\varphi_0}\langle\delta\dot{\varphi}\phi\rangle \\
&- \left. \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle \right\} \quad,
\end{aligned}
\tag{4}
$$

where H is the Hubble expansion rate and the $\langle\rangle$ denote spatial averaging.

Specializing to the case of slow-roll inflation (with φ as the inflaton) and focusing on the effects of long wavelength or IR modes (modes with wavelength larger than the Hubble radius), the EMT simplifies to

$$
\tau_0^0 \cong \left(2\frac{V''V^2}{V'^2} - 4V \right) <\phi^2> \cong \frac{1}{3}\tau_i^i,
\tag{5}
$$

and

$$
p \equiv -\frac{1}{3}\tau_i^i \cong -\tau_0^0.
\tag{6}
$$

so that $\rho_{eff} < 0$ with the equation of state $\rho = -p$.

The factor $\langle\phi^2\rangle$ is proportional to the IR phase space so that, given a sufficiently long period of inflation (in which the phase space of super-Hubble modes grows continuously), τ_0^0 can become important and act to cancel any positive energy density (i.e. as associated with the inflaton, or a cosmological constant) and bring inflation to an end - a natural graceful exit, applicable to any model in which inflation proceeds for a sufficiently long time.

Due to this behaviour during inflation, it was speculated [15] that this could also be used as a mechanism to relax the cosmological constant, post-reheating - a potential solution to the old cosmological constant problem. However, this mechanism works (if at all - see this discussion in the concluding section) only if inflation lasts for a very long time (if the potential of φ is quadratic, the condition is that the initial value of φ is larger than $m^{-1/3}$ in Planck units).

III. BEYOND SLOW-ROLL

Here, we will ask the question what role back-reaction of IR modes can play in those models of inflation in which inflation ends naturally (through the reheating dynamics of φ) before the phase space of long wavelength modes has time to build up to a dominant value. In order to answer this question, we require an expression for $\tau_{\mu\nu}$ unfettered by the slow-roll approximation. Doing this provides us with an expression for the EMT which is valid during preheating and, more importantly, throughout the remaining course of cosmological evolution.

In the long wavelength limit, we have [34],

$$
\tau_{00} = \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle,
\tag{7}
$$

and

$$
\begin{aligned}
\tau_{ij} &= a^2\delta_{ij}\left\{ \frac{1}{8\pi G}\left[(24H^2 + 16\dot{H})\langle\phi^2\rangle \right] + 4\dot{\varphi_0}^2\langle\phi^2\rangle \right\} \\
&- \frac{1}{2}V''(\varphi_0)\langle\delta\varphi^2\rangle + 2V'(\varphi_0)\langle\phi\delta\varphi\rangle \right\}.
\end{aligned}
\tag{8}
$$

As in the case of slow-roll, we can simplify these expressions by making use of the constraint equations which relate metric and matter fluctuations [13], namely

$$
-(\dot{H} + 3H^2)\phi \simeq 4\pi G V' \delta\varphi.
\tag{9}
$$

Then, (7) and (8) read

$$\tau_{00} = (2\kappa^2 \frac{V''}{(V')^2}(\dot{H}+3H^2)^2 - 4\kappa(\dot{H}+3H^2))\langle\phi^2\rangle,$$
(10)

$$\tau_{ij} = a^2\delta_{ij}(12\kappa(\dot{H}+H^2)+4\dot{\varphi_0(t)}^2 - 2\kappa^2\frac{V''}{(V')^2}(\dot{H}+3H^2)^2)\langle\phi^2\rangle,$$
(11)

with $\kappa = \frac{M_{Pl}^2}{8\pi}$.

The above results are valid for all cosmological eras. With this in mind, we now turn an eye to the post-inflation universe and see what the above implies about its subsequent evolution.

In what follows, we take the scalar field potential to be $\lambda\varphi^4$. As was shown in [16], the equation of state of the inflaton after reheating is that of radiation, which implies $\varphi(t) \propto 1/a(t)$.

IV. THE RADIATION EPOCH

The radiation epoch followed on the heels of inflation. The EMT in this case reads

$$\tau_{00} = (\frac{1}{16}\kappa^2\frac{V''}{(V')^2}\frac{1}{t^4} - \frac{\kappa}{t^2})\langle\phi^2\rangle,$$
(12)

$$\tau_{ij} = a^2(t)\delta_{ij}(-3\frac{\kappa^2}{t^2}+4(\dot{\varphi})^2 - \frac{1}{16}\kappa^2\frac{V''}{(V')^2}\frac{1}{t^4})\langle\phi^2\rangle.$$ (13)

The first thing we notice is that, if the time dependence of $\langle\phi^2\rangle$ is negligible, the EMT acts as a tracker with every term scaling as $1/a^4(t)$ (except for the $\dot{\varphi}$ which scales faster and which we ignore from now on).

We now determine the time dependence of $\langle\phi^2\rangle$, where

$$\langle\phi^2\rangle = \frac{\psi^2}{V}\int d^3\vec{x}\, d^3\vec{k_1}\, d^3\vec{k_2}\, f(\vec{k_1})f(\vec{k_2})e^{i(\vec{k_1}+\vec{k_2})\cdot\vec{x}},$$ (14)

with

$$f(\vec{k}) = \sqrt{V}(\frac{k}{k_n})^{-3/2-\xi}k_n^{-3/2}e^{i\alpha(\vec{k})}.$$
(15)

Here, ψ represents the amplitude of the perturbations (which is constant in time), ξ represents the deviation from a Harrison-Zel'dovich spectrum, $\alpha(\vec{k})$ is a random variable, and k_n is a normalization scale.

Taking $\frac{\Lambda}{a(t)}$ as a time-dependent, infra-red cutoff and the Hubble scale as our ultra-violet cutoff, and focusing on the case of a nearly scale-invariant spectrum, the above simplifies to

$$\langle\phi^2\rangle = 4\pi\psi^2 k_n^{-2\xi}\int_{\frac{\Lambda}{a(t)}}^{H} dk_1\frac{1}{k_1^{1-2\xi}}$$
(16)

(17)

In the limit of small ξ, the above reduces to

$$\langle\phi^2\rangle \cong 4\pi\psi^2\ln(\frac{a(t)H}{\Lambda}).$$
(18)

The time variation of the above quantity is only logarithmic in time and hence not important for our purposes. As well, given the small amplitude of the perturbations, $\langle\phi^2\rangle \ll 1$. Note that this condition is opposite to what needs to happen in the scenario when gravitational back-reaction ends inflation.

Now that we have established that the EMT acts as a tracker in this epoch, we still have to determine the magnitude of τ_{00} and the corresponding equation of state. In order to do this, as in [16], we assume that the preheating temperature is $T = 10^{12}$ GeV, the quartic coupling $\lambda = 10^{-12}$, and the inflaton amplitude following preheating is $\varphi_0 = 10^{-4}M_{Pl}$. Making use of

$$a(t) = (\frac{32\pi\rho_0}{3M_{Pl}^2})^{1/4}t^{1/2},$$
(19)

where ρ_0 is the initial energy density of radiation, we find

$$\tau_{00} = -\kappa(\frac{32\pi\rho_0}{3M_{Pl}^2})\frac{1}{a^4(t)}[1-\frac{1}{8}]\langle\phi^2\rangle \cong -\frac{4}{3}\frac{\rho_0}{a^4(t)}\langle\phi^2\rangle,$$
(20)

$$\tau_{ij} = -a^2(t)\delta_{ij}\kappa(\frac{32\pi\rho_0}{3M_{Pl}^2})\frac{1}{a^4(t)}[3+\frac{1}{8}]\langle\phi^2\rangle \cong -4\frac{\rho_0}{a^4(t)}\langle\phi^2\rangle.$$
(21)

We find that, as in the case of an inflationary background, the energy density is negative. However, unlike during inflation, the equation of state is no longer that of a cosmological constant. Rather, $w \cong 3$. Clearly, due to the presence of $\langle\phi^2\rangle$, this energy density is sub-dominant. Using the value of ψ in (16) determined by the normalization of the power spectrum of linear fluctuations from CMB experiments [17], we can estimate the magnitude to be approximately four orders of magnitude below that of the cosmic fluid. Any observational constraints that could arise during the radiation era (e.g. from primordial nucleosynthesis, or the CMB) will hence be satisfied.

V. MATTER DOMINATION

During the period of matter domination, we find that the EMT reduces to

$$\tau_{00} = (\frac{2}{3}\frac{\kappa^2}{\lambda}\frac{a^4(t)}{\varphi^4}\frac{1}{t^4} - \frac{8}{3}\frac{1}{t^2})\langle\phi^2\rangle.$$
(22)

$$\tau_{ij} = (-\frac{2}{3}\frac{\kappa^2}{\lambda}\frac{a^4(t)}{\varphi^4}\frac{1}{t^4} - \frac{8}{3}\frac{1}{t^2})\langle\phi^2\rangle.$$
(23)

In arriving at these equations, we are assuming that the matter fluctuations are carried by the same field φ (possibly the inflaton) as in the radiation epoch, a field which

scales in time as $a^{-1}(t)$ [35]. This result is quite different from what was obtained in the radiation era for the following reason: previously, we found that both terms in τ_{00} scaled in time the same way. Now, we find (schematically)

$$\tau_{00} \propto \frac{\kappa^2}{a^2(t)} - \frac{\kappa}{a^3(t)}. \qquad (24)$$

The consequences of this are clear: the first term will rapidly come to dominate over the second, which is of approximately the same magnitude at matter-radiation equality. This will engender a change of sign for the energy density and cause it to eventually overtake that of the cosmic fluid. The same scaling behaviour is present in τ_{ij} and so the equation of state of the EMT will rapidly converge to that of a cosmological constant, but this time one corresponding to a positive energy density.

Matter-radiation equality occurred at a redshift of about $z \approx 10^4$ and we find that

$$\tau_{00}(z=0) \simeq \rho_m(z=0), \quad w \simeq -1, \qquad (25)$$

and thus we are naturally led to a resolution of the both aspects of the dark energy problem. We have an explanation for the presence of a source of late-time acceleration, and a natural solution of the "coincidence" problem: the fact that dark energy is rearing its head at the present time is directly tied to the observationally determined normalization of the spectrum of cosmological perturbations.

VI. DARK ENERGY DOMINATION AND INFLATON BACK-REACTION

Does this model predict that, after an initial stage of matter domination, the universe becomes perpetually dominated by dark energy? To answer this question, one needs to examine the effects of back-reaction on the late time scalar field dynamics.

The EMT predicts an effective potential for φ that differs from the simple form we have been considering so far. During slow-roll, we have that

$$V_{eff} = V + \tau_0^0. \qquad (26)$$

One might expect that this would lead to a change in the spectral index of the power spectrum or the amplitude of the fluctuations. To show that this is not the case, we can explicitly calculate the form of V_{eff} for the case of an arbitrary polynomial potential and see that, neglecting any φ dependence of $\langle\phi^2\rangle$, (26) implies an (a priori small) renormalization of the scalar field coupling. We find that the inclusion of back-reaction does not lead to any change in the spectral index (in agreement with [18]) or to any significant change in the amplitude of the perturbations.

During radiation domination, we find that the ratio of $\frac{\tau_0^0}{V}$ is fixed and small, so that scalar field back-reaction does not play a significant role in this epoch. In fact, back-reaction on the scalar field does not become important until back-reaction begins to dominate the cosmic energy budget. In that case,

$$V_{eff} \sim \frac{1}{\varphi^4}, \qquad (27)$$

causing the φ to "roll up" it's potential. Once φ comes to dominate, the form of the effective potential changes to

$$V_{eff} \sim \varphi^4, \qquad (28)$$

and φ immediately rolls down it's potential, without the benefit of a large damping term (given by the Hubble scale).

Thus, this model predicts alternating periods of dark energy/matter domination, which recalls the ideas put forth in [15].

From the point of view of perturbation theory, we see that in the regime where the higher-order terms begin to dominate and the series would be expected to diverge, these corrections are then suppressed and become subdominant again.

VII. DISCUSSION AND CONCLUSIONS

To recap, we find that, in the context of inflationary cosmology, the EMT of long wavelength cosmological perturbations can provide a candidate for dark energy which resolves the "new cosmological constant" (or "coincidence" problem in a natural way. Key to the success of the mechanism is the fact that the EMT acts as a tracker during the period of radiation domination, but redshifts less rapidly than matter in the matter era. The fact that our dark energy candidate is beginning to dominate today, at a redshift 10^4 later than at the time of equal matter and radiation is related to the observed amplitude of the spectrum of cosmological perturbations.

We wish to conclude by putting our work in the context of other recent work on the gravitational back-reaction of cosmological perturbations. We are making use of non-gradient terms in the EMT (as was done in [8, 9]). As was first realized by Unruh [19] and then confirmed in more detail in [20, 21], in the absence of entropy fluctuations, the effects of these terms are not locally measurable (they can be undone by a local time reparametrization). It is important to calculate the effects of back-reaction on local observables measuring the expansion history. It was then shown [22] (see also [23]) that in the presence of entropy fluctuations, back-reaction of the non-gradient terms is physically measurable, in contrast to the statements recently made in [24] [36]. In our case, we are making use of fluctuations of the scalar field φ at late times. As long as this fluctuation is associated with an isocurvature mode, the effects computed in this paper using the EMT approach should also be seen by local observers.

Our approach of explaining dark energy in terms of back-reaction is different from the proposal of [26]. In that approach, use is made of the leading gradient terms in the EMT. However, it has subsequently been shown [27] that these terms act as spatial curvature and that hence their magnitude is tightly constrained by observations. Other criticism was raised in [28] where it was claimed that, in the absence of a bare cosmological constant, it is not possible to obtain a cosmology which changes from deceleration to acceleration by means of back-reaction. This criticism is also relevant for our work. However, as pointed out in [29], there are subtleties when dealing with spatially averaged quantities, even if the spatial averaging is over a limited domain, and that the conclusions of [28] may not apply to the quantities we are interested in.

There have also been attempts to obtain dark energy from the back-reaction of short wavelength modes [30–32]. In these approaches, however, nonlinear effects are invoked to provide the required magnitude of the back-reaction effects.

We now consider some general objections which have been raised regarding the issue of whether super-Hubble-scale fluctuations can induce locally measurable back-reaction effects. The first, and easiest to refute, is the issue of causality. Our formalism is based entirely on the equations of general relativity, which are generally covariant and thus have causality built into them. We are studying the effects of super-Hubble but sub-horizon fluctuations [37]. Another issue is locality. As shown in [33], back-reaction effects such as those discussed here can be viewed in terms of completely local cosmological equations. For a more extensive discussion, the reader is referred to [25].

In conclusion, we have presented a model which can solve the dark energy problem without resorting to new scalar fields, making use only of conventional gravitational physics. The effect of the back-reaction of the super-Hubble modes is summarized in the form of an effective energy-momentum tensor which displays distinct behaviour during different cosmological epochs.

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[34] We've ignored terms proportional to $\dot{\phi}$ on the basis that such terms are only important during times when the equation of state changes. Such changes could lead to large transient effects during reheating but would be negligible during the subsequent history of the universe.

[35] Even if we were to add a second scalar field to represent the dominant matter and add a corresponding second matter term in the constraint equation (9), it can be seen that the extra terms in the equations for the effective EMT decrease in time faster than the dominant term discussed here.

[36] There are a number of problems present in the arguments of [24], in addition to this point. We are currently preparing a response that addresses the criticisms of these authors. See [25].

[37] We remind the reader that it is exactly because inflation exponentially expands the horizon compared to the Hubble radius that the inflationary paradigm can create a causal mechanism for the origin of structure in the universe. In our back-reaction work, we are using modes which, like those which we now observe in the CMB, were created inside the Hubble radius during the early stages of inflation, but have not yet re-entered the Hubble radius in the post-inflationary period.